



**CCUS: 4014548**

## **Assessing the Applicability of Analytical Equations for Predicting CO<sub>2</sub> Pressure Build-Up and Plume Migration for Real Geological Conditions**

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Copyright 2024, Carbon Capture, Utilization, and Storage conference (CCUS) DOI 10.15530/ccus-2024-4014548

This paper was prepared for presentation at the Carbon Capture, Utilization, and Storage conference held in Houston, TX, 11-13 March.

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### **Abstract**

The sequestration of carbon dioxide (CO<sub>2</sub>) in subsurface geological formations is one of the foremost techniques to combat climate change attributed to CO<sub>2</sub> emissions. Among potential reservoirs, deep saline aquifers emerge as particularly promising due to their substantial pore volume, albeit subject to stringent criteria to ensure enduring storage without seepage into freshwater aquifers or the atmosphere. In this study, we investigate the applicability of analytical equations for estimating the pressure buildup in an aquifer due to injection. We compared analytical results to the results from numerical reservoir simulations to compare the difference in results. Along with this, we conducted a sensitivity analysis to understand under which reservoir conditions the analytical equation does not apply. Results demonstrated that the analytical equation is adequate to estimate pressure buildup for long injection times with distances close to the injection well. Conversely, it is established that the analytical equation proves ineffective in scenarios featuring low aquifer permeability or thickness, as well as instances where the injected CO<sub>2</sub> lacks liquid-like density. These findings are important in verifying the applicability of the analytical equation for quick estimates of pressure in the site selection process.

### **Introduction**

The need for underground carbon dioxide (CO<sub>2</sub>) storage arises from the imperative to mitigate the impacts of anthropogenic greenhouse gas emissions on climate change. As human activities, particularly the burning of fossil fuels, continue to release significant amounts of CO<sub>2</sub> into the atmosphere, there is a growing urgency to reduce these emissions. Underground CO<sub>2</sub> storage, also known as carbon capture and storage (CCS), provides a viable solution by capturing CO<sub>2</sub> emissions at their source, such as power plants and industrial facilities, and storing them deep underground in geological formations. This process

prevents the released CO<sub>2</sub> from entering the atmosphere and contributing to the greenhouse effect. By securely sequestering CO<sub>2</sub> underground, CCS plays a crucial role in achieving emission reduction targets and transitioning towards a more sustainable and low-carbon future. Additionally, it allows for the continued use of fossil fuels in the short to medium term while facilitating the transition to cleaner energy sources.

A critical factor concerning the storage capacity of a geological formation is the resulting pressure buildup from injected CO<sub>2</sub>. When assessing the suitability of a given formation, it is imperative to verify that the estimated pressure buildup does not exceed the mechanical integrity of the formation seal. Calculating the pressure buildup requires simulating the injection of supercritical CO<sub>2</sub> into the formation. This is usually achieved with numerical multi-phase compositional reservoir simulators. These models can be extremely accurate in modeling reservoir conditions, however, are often computationally intensive. An alternative to calculate pressure buildup without numerical simulation is with analytical solutions. Analytical solutions aim to rapidly calculate the pressure buildup during injection by making multiple assumptions about the flow behavior in the reservoir.

One prominent analytical solution, proposed by Benson (2003), encapsulates the physical processes inherent in the injection of CO<sub>2</sub> into water. This methodology accounts for factors such as relative permeability effects, capillary pressure effects, adverse mobility ratio, pressure and temperature dependencies of CO<sub>2</sub> density and viscosity, as well as the partitioning of CO<sub>2</sub> and water phases. Assumptions include a Buckley-Leverett type displacement, vertical equilibrium in a 2D horizontal reservoir, homogeneity of the reservoir, full penetration of the reservoir, infinite acting radial flow, and a slightly compressible fluid composition. This analytical model has only been verified against the TOUGH2 case presented by Pruess et al. (2001). The verification did not examine in detail the limits of reservoir and geological parameters to understand if the analytical equation can suffice for all reservoir properties. Similarly, the impact of the assumptions, especially reservoir heterogeneity and partial penetration of the reservoir, has not been examined. Extending the verification to encompass various numerical cases is essential for comprehending the applicability limits of this analytical solution. Thus, the primary focus of this study lies in the verification of Benson's solution through a comparative analysis with multiple numerical cases, coupled with sensitivity analyses.

### List of symbols

$b$	Formation thickness [m <sup>2</sup> ]
$c_t$	Total compressibility [pa <sup>-1</sup> ]
$f_{CO_2}$	Fractional flow of CO <sub>2</sub>
$k$	Permeability [m <sup>2</sup> ]
$k_{rCO_2}$	CO <sub>2</sub> Relative permeability
$q_{CO_2}$	Injection rate [m <sup>3</sup> /day]
$r_f$	Radial extent of CO <sub>2</sub> front [m]
$r_w$	Investigation (well) radius [m]
$s_{CO_2}$	Average saturation behind CO <sub>2</sub> front
$t$	Time of injection [seconds]
$\mu_{CO_2}$	CO <sub>2</sub> viscosity [pa-s]
$\mu_w$	Water viscosity [pa-s]
$\rho_{CO_2}$	CO <sub>2</sub> density [kg/m <sup>3</sup> ]

## Methods

The analytical solution developed by Benson (2003) for pressure buildup considers both a steady-state pressure buildup term behind the CO<sub>2</sub> front and a transient pressure buildup term outside the front. The formulation of the approximate solution is expressed as follows:

$$\Delta p(r_w, t) = \Delta p_{s.s.}(r_w, t) + \Delta p_t(r_f, t) \quad (1)$$

where the steady state pressure buildup behind the saturation front is given by:

$$\Delta p_{s.s.}(r_w, t) = \frac{q_{CO_2} \bar{\mu}_{CO_2}}{2\pi \bar{\rho}_{CO_2} kb} \left[ \ln \frac{r_f}{r_w} + \left( \frac{f_{CO_2}}{k_{rCO_2}} \Big|_{r_f} - 1 \right) * \left( 1 - \frac{r_w}{r_f - r_w} \ln \frac{r_f}{r_w} \right) \right] \quad (2)$$

and the transient pressure buildup outside the front is given by the following:

$$\Delta p_t(r_f, t) = \frac{q_{CO_2} \mu_w}{4\pi \bar{\rho}_{CO_2} kb} W(u_f) \quad (3)$$

$$W(u_f) \cong -0.5772 - \ln \left( \frac{\phi \mu_w c_t r_f^2}{4kt} \right) \quad (4)$$

$$r_f = \left( \frac{q_{CO_2} t}{\pi b \phi \bar{s}_{CO_2}} \right)^{\frac{1}{2}} \quad (5)$$

The pressure buildup (Equation 1) is a composite function of both the steady state (Equation 2) and the transient (Equation 3) terms. It is anticipated that the viscosity and density of the CO<sub>2</sub> will change as a function of both distance from the well and time, due to the changing pressures and temperatures resulting from the injection of supercritical CO<sub>2</sub>. Consequently, the analytical equation relies on the utilization of average values for fluid properties behind the saturation front.

Leveraging this analytical equation, we constructed and analyzed a singular injection scenario for a theoretical aquifer model. Our injection scenario was 1 Mt/yr of CO<sub>2</sub> over a duration of 3 years. Commencing with the verification of the analytical equation, our approach started with establishing a base case scenario for reservoir and fluid properties, seen in Table 1 below.

**Table 1 - Base case reservoir and fluid properties.**

Property	Value	Unit
b	150	m
c <sub>t</sub>	1.01E-09	1/pa
f <sub>CO<sub>2</sub></sub>	0.847361	
k	50	mD
k <sub>rCO<sub>2</sub></sub>	0.06841	
q <sub>CO<sub>2</sub></sub>	1	Mt/yr
r <sub>f</sub>	450.01	m
s <sub>CO<sub>2</sub></sub>	0.252381	
t	3	years
μ <sub>CO<sub>2</sub></sub>	4.73E-05	pa-s

$\mu_w$	0.000573	pa-s
$\rho_{CO_2}$	622.81	kg/m <sup>3</sup>
$\phi$	0.2	

Along with this, Figures 1 and 2 below depict the relative permeability as well as fractional flow curves used to model fluid properties, respectively.

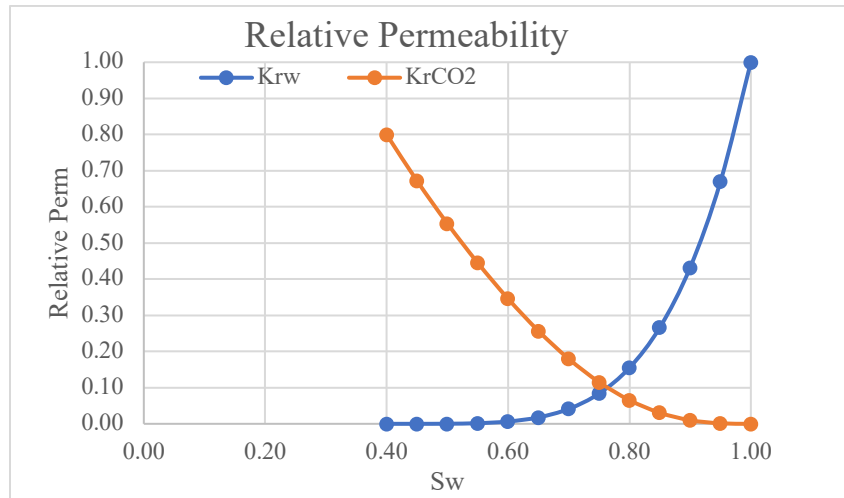


Figure 1 - Relative permeability curves used for this study.

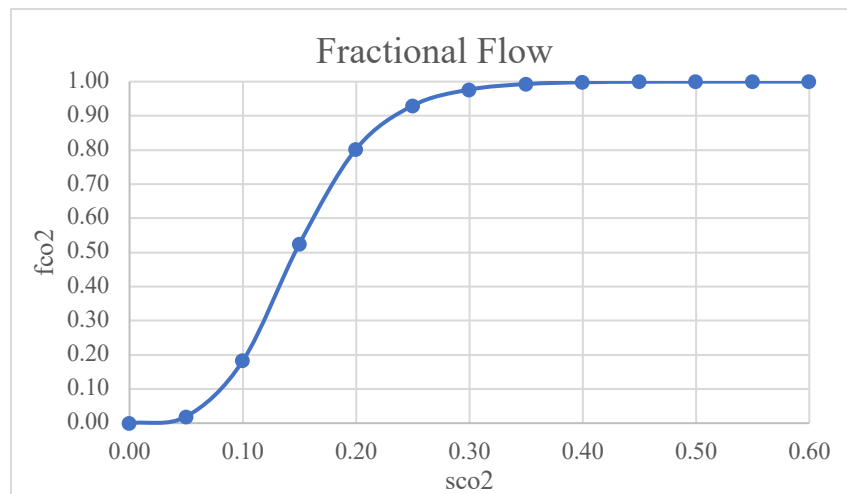
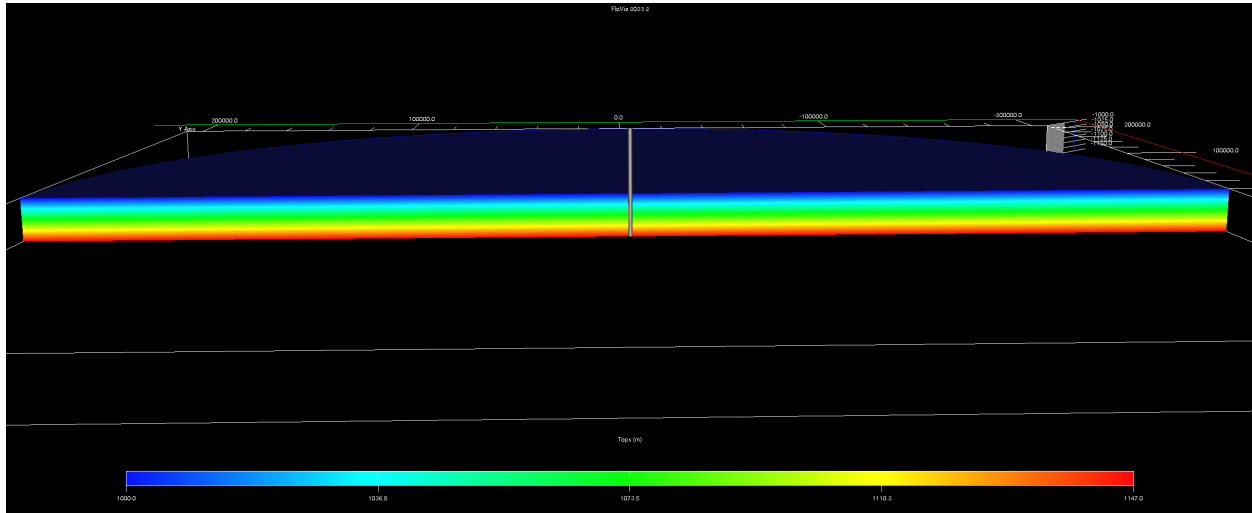


Figure 2 - CO2 Fractional Flow Curve for this study.

Numerical Model Development:

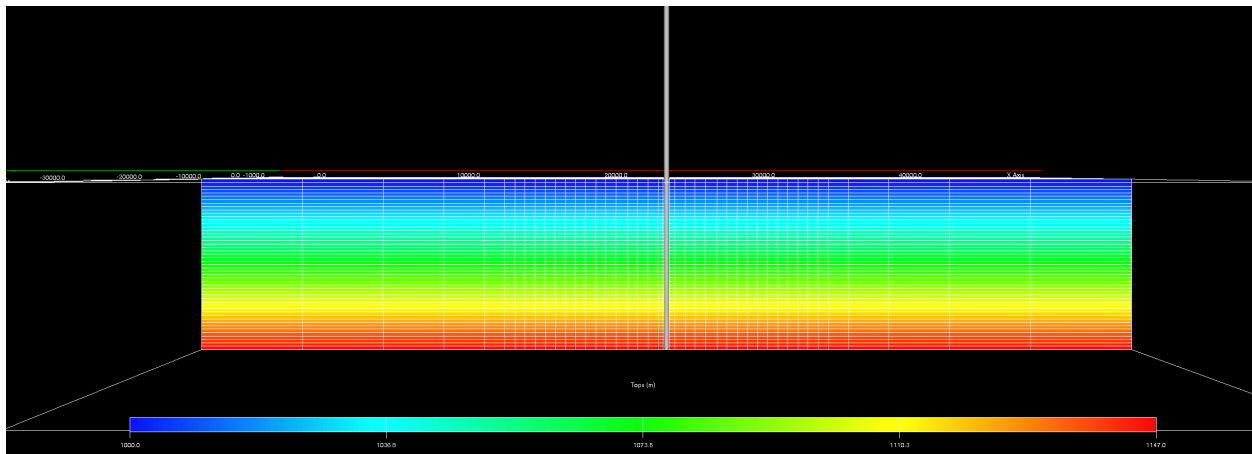
Upon completion of the pressure buildup calculations using the analytical equation, we then moved towards developing a representative model using a commercial reservoir simulator. The initial model designed was a cylindrical model. This was done to comply with the flow assumptions used in the analytical equation. The cylindrical model consisted of 20,000 blocks separated in 50 equal layers, logarithmically increasing in size, spanning a total radius of 200,000 meters. The intention behind this configuration was to create an infinite-acting aquifer model, facilitating the dissipation of pressure.

Notably, this model, depicted in Figure 3 below, incorporated identical properties as those employed in the base analytical model.



**Figure 3 - Cylindrical model slice, showing layer tops (m).**

In tandem with the cylindrical model, we constructed a rectangular model, aligning with conventional practices and offering flexibility in accommodating varying reservoir properties. The rectangular model that we created consisted of 130,050 cells distributed across 50 layers. This model, shown below in Figure 4, expands to around 25,0000 meters away from the injection well on all sides.



**Figure 4 - Rectangular model slice, showing layer tops (m).**

Analysis of both the cylindrical and rectangular base cases verified that either model generated the same pressure buildup. As a result, only the cylindrical model was used for sensitivities to reduce computational time. To gain an insight into how accurate the analytical equation models the pressure buildup compared to a numerical simulator, we ran multiple sensitivities. The parameters we altered included permeability, thickness, temperature, and initial pressure. Table 2 below depicts the sensitivity properties and their values.

**Table 2 - Sensitivity parameters, with base case underlined.**

Property	Values						
k (mD)	10	<u>50</u>	100	250	500	1000	
b (m)	9	30	60	90	<u>150</u>	210	
T (C)	42.5	<u>47.5</u>	52.5	57.5			
P (bar)	40	80	<u>120</u>	160	200	240	280

The objective of the sensitivity analysis was to discern the trends in the impact of various parameters on the pressure buildup equation. A multifaceted approach was employed in the study's analysis, incorporating techniques such as value differentiation, the creation of cross plots comparing numerical and analytical solutions, and the visualization of pressure buildup over time and distance through plotted graphs. These methods were integrated to understand and interpret the relationships between the altered parameters and the resulting behavior of the pressure buildup equation.

## Results

For each of the 20 cases trialed, a dedicated numerical model was created to simulate the pressure buildup. The results of the pressure buildup (in bar) for each case are presented in Tables 3 and 4, detailing the outcomes obtained through both the analytical and numerical solutions, respectively.

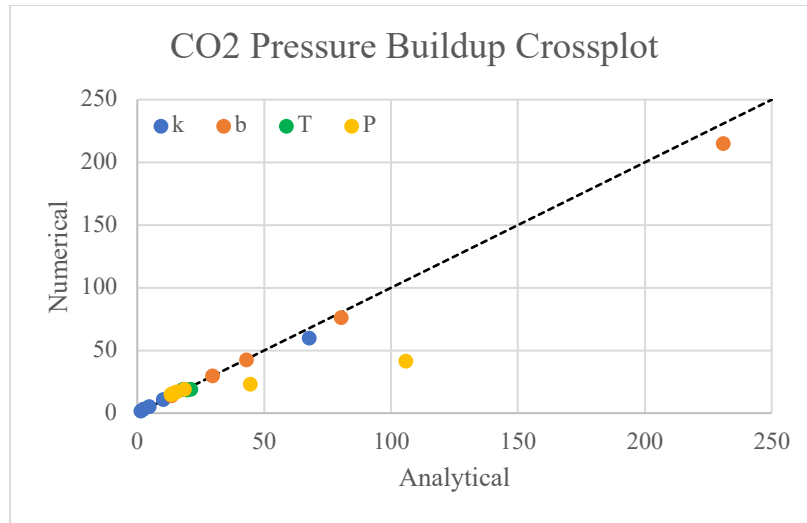
**Table 3 - Analytical solution pressure buildup (bar)**

Analytical							
k	67.846	18.544	10.343	4.704	2.566	1.390	
b	231.102	80.460	43.060	29.722	18.544	13.555	
T	18.125	18.544	19.610	21.127			
P	105.978	44.597	18.544	15.637	14.486	13.773	13.258

**Table 4 - Numerical solution pressure buildup (bar)**

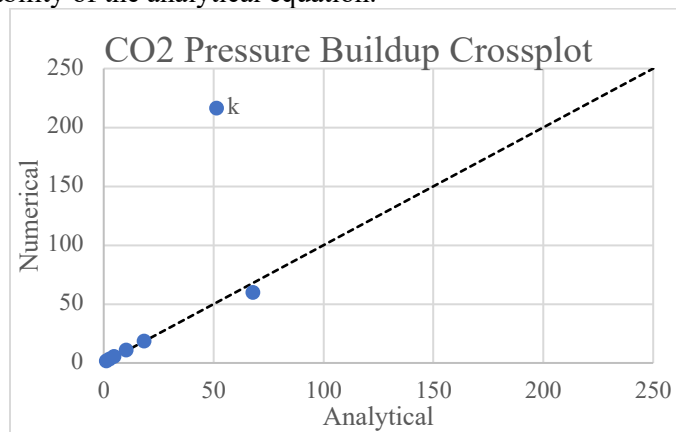
Numerical							
k	60.026	18.841	10.995	5.332	3.114	1.879	
b	214.812	76.159	42.335	29.579	18.841	13.928	
T	19.087	18.841	18.752	18.945			
P	41.2382	22.993	18.841	17.068	15.918	15.177	14.698

To visualize the overall effect of how the properties alter the pressure buildup, we created a cross plot of the analytical and numerical solutions, shown below in Figure 5.

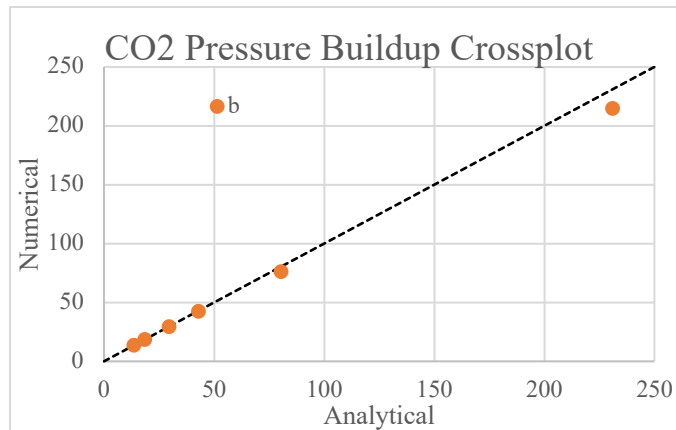


**Figure 5 - Sensitivity analysis pressure buildup cross plot for all cases.**

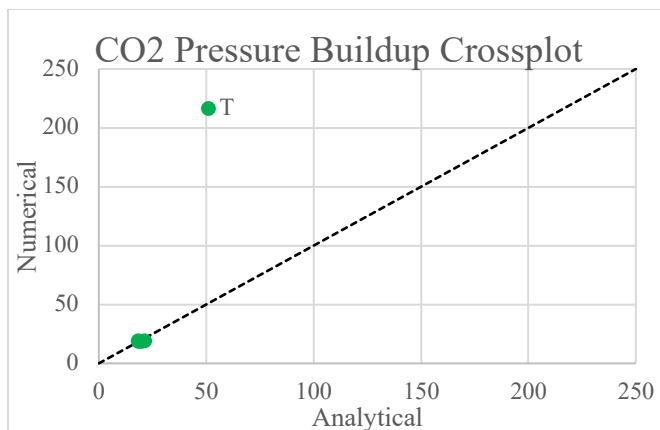
Along with the overall effect, cross plots of each individual property can be found in Figures 6 through 9. The generation of these cross plots was useful in determining the limits of different properties when determining the applicability of the analytical equation.



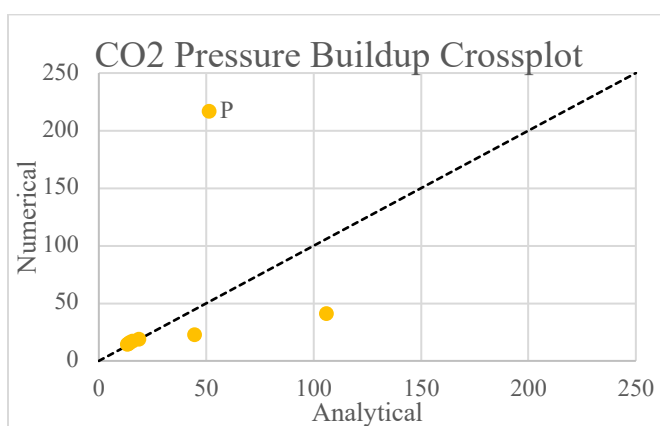
**Figure 6 - Permeability sensitivity cross plot.**



**Figure 7 - Thickness sensitivity cross plot.**



**Figure 8 - Temperature sensitivity cross plot.**



**Figure 9 - Initial pressure sensitivity cross plot.**

During the analysis of the cross plots comparing the analytical and numerical solutions, the anticipated outcome was the formation of a unit slope line, indicating overlap between the points and signifying equivalence between the analytical and numerical solutions for each case. Any deviation from this line would suggest that the property value for that particular case falls outside the applicable range of the analytical equation. The cross plots generated for our 20 sensitivity cases exhibited a favorable distribution, aligning closely with the trendline, with a few exceptions.

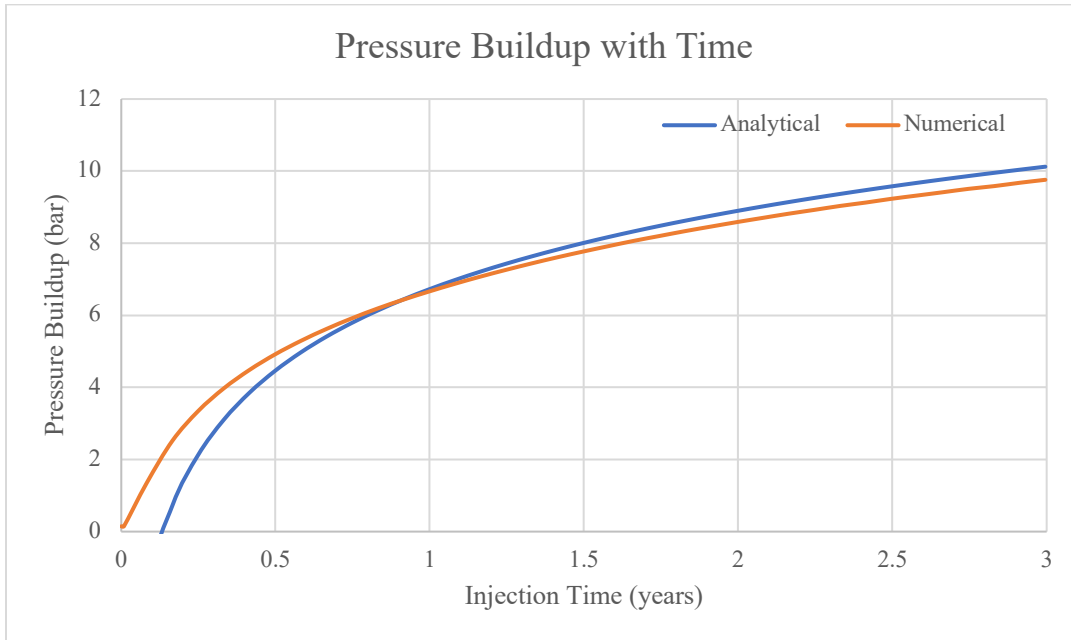
Notably, the deviation observed in the case of the lowest permeability sensitivity (10 mD) may not raise significant concerns, as such low permeability levels would likely render the aquifer unsuitable for CO<sub>2</sub> sequestration, minimizing the impact of the slight analytical solution discrepancy. Similarly, the analytical solution's deviation in the case of the lowest thickness sensitivity (9 meters) is of limited concern, given that such thin reservoirs would typically be disqualified during the site selection process, as determined by Callas et. al (2022).

Another noteworthy deviation occurred in the cases with the two lowest initial pressures (40 and 80 bar), where the analytical equations failed to accurately represent the behavior. This discrepancy can be attributed to the fact that, at the given reservoir temperature, CO<sub>2</sub> in the 40 bar cases exists solely in the vapor phase, while the 80-bar case is supercritical but exhibits vapor-like density. The analytical solution assumes a supercritical, liquid-like density for CO<sub>2</sub>, leading to the expected failure in these cases. However, given that such initial pressures would likely result in disqualification during the site selection process, these deviations do not significantly impact the overall findings.

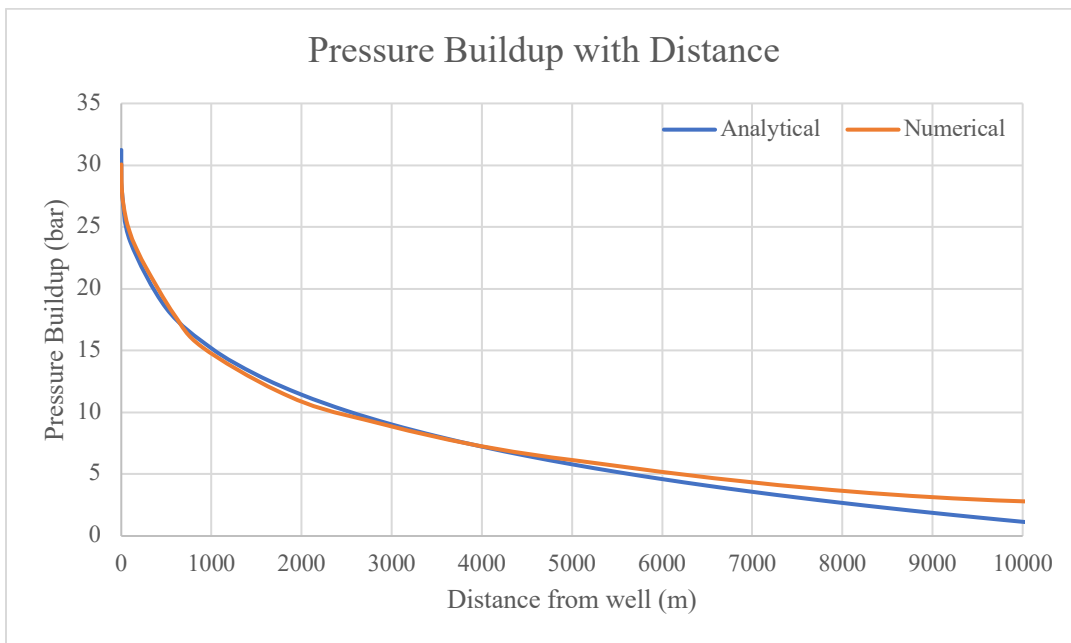


Excluding the mentioned cases, the cross plots of analytical and numerical solutions demonstrate a satisfactory trend, instilling confidence in the use of the analytical equations for estimating pressure buildup across a range of scenarios.

In conjunction with the cross plots, we generated graphs illustrating the pressure buildup over time (Figure 10) and distance (Figure 11) for both the analytical and numerical base cases. The purpose of these plots is to understand in which ranges of distance and time that the analytical equation remains applicable. This is because some of the assumptions made in the development of the analytical equation are long injection periods and pressure buildup at short distances.



**Figure 10 - Pressure buildup with time.**



**Figure 11 - Pressure buildup with distance.**

The analysis of pressure buildup over time presented in Figure 10 aligns with expectations. The analytical equation, under the assumption of long injection periods, naturally exhibits significant deviations from the numerical solutions for times below approximately 0.75 years. Beyond this initial period, however, a notable convergence is observed between the analytical and numerical solutions, with minimal error (<2%). The exceptionally low error in this later phase supports the assertion that the analytical equation accurately predicts pressure buildup for extended injection periods. Consequently, it is reasonable to conclude that the analytical equation provides reliable estimations for pressure buildup over time during prolonged injection scenarios.

Figure 11, depicting pressure buildup over distance, reveals outcomes consistent with the observations made in the time analysis. The analytical equation employed assumes the calculation of pressure buildup for close distances. From the figure, it is evident that the analytical and numerical models yield very similar solutions up to approximately 5000 meters from the well. In the context of pressure buildup calculations, the primary objective is often to ascertain the highest pressure at the wellbore, ensuring that fractures are not induced. For the base case illustrated in this figure, the percentage difference between the models at the injection well remains minimal, once again below 2%. This negligible error observed over distance supports the conclusion that the analytical equation is suitable for estimating pressure buildup within relatively short distances from the injection well. The agreement between analytical and numerical solutions in this context underscores the adequacy of the analytical approach for predicting pressure buildup in the near vicinity of the injection well.

## Conclusions

In this study, we systematically explored the applicability of analytical equations for predicting the pressure buildup resulting from CO<sub>2</sub> injection into a saline aquifer. Employing reservoir simulation and sensitivity analyses, we aimed to discern the conditions under which the analytical equation yields accurate results.

Our investigation revealed that the analytical equation is well-suited for estimating pressure buildup at short distances over extended injection periods, contingent upon the aquifer meeting specific criteria for CO<sub>2</sub> sequestration. Notably, the analytical equation proved inadequate when applied to scenarios characterized by low permeability (<10 mD), low thickness (<9 meters), and non-supercritical or liquid-like densities influenced by varying pressure and temperatures.

Subsequently, we determined that the analytical equation is adequate in estimating pressure buildup within most aquifers that have favorable reservoir properties for CO<sub>2</sub> storage. This verification of the analytical equation presented supports the original author's claim that the equation can be used to quickly provide estimates for pressure buildup in saline aquifers. This is important because it allows for quick ranking of sites based on whether the estimated pressure buildup exceeds fracture pressure for a certain formation.

## References

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- Pruess, K., Doughty, C., Benson, S. et al. 2001. Capacity Investigation of Brine-Bearing Sands of the Frio Formation for Geologic Sequestration of CO<sub>2</sub>. Paper presented at the First National Conference on Carbon Sequestration, Washington, DC, USA, 14-17 May.

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